B.Sc.

MATHAMETICS

I Semester End Examination- March/April 2022

ALGEBRA-I and CACULUS-I

Course Code: MAT1DSC01 **QP code:1015**

Time: 2hrs **Max. Marks: 60**

Instruction to Candidates: Answer all questions

I. Answer any Six of the following Questions: 2 x 6 = 12

1. Define skew-symmetric matrix. Give an example.
2. If λ is an eigen value of a square matrix A with X as a corresponding eigen vector, then show that λ^2 is the eigen value of A^2
3. Find n^{th} derivative $e^{3x}\ cos4x$
4. Show that does not exist for $\lim_{x\to 0}\left(\frac{1}{1-e^{\frac{1}{x}}}\right)$
5. State Rolle's mean value theorem.
6. Evaluate using L' Hospital's rule.

$$\lim_{x\to 0} \frac{a^x - b^x}{x}$$

7. If $u = x^3 + 3x^2y$ then show that $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}$
8. Find total derivative $\frac{du}{dt}$ of $u = xy$, where $x = \log_e t\,, y = e^t$

II. Answer any Two of the following Questions: 2 x 6 = 12

1. Find the rank of the matrix $\begin{pmatrix} 1 & 2 & 3 & 2 \\ 2 & 3 & 5 & 1 \\ 1 & 3 & 4 & 5 \end{pmatrix}$ by reducing it to echelon form.
2. Test the following system of equations for consistency and solve them if they are consistent

$$x + 2y - z = 3,$$
$$3x - y + 2z = 1,$$
$$2x - 2y + 3z = 2$$

3. a) By using Cayley-Hamilton Theorem, find the inverse of the matrix $A = \begin{bmatrix} 1 & 2 \\ 5 & 4 \end{bmatrix}$

   b) Find the Eigen values of the matrix $\begin{bmatrix} 1 & 2 & -3 \\ 0 & 2 & 1 \\ 0 & 0 & -3 \end{bmatrix}$

III. Answer any Six of the following Questions: 6 x 6 = 36

1. a) Discuss the differentiability of the function $f(x) = \begin{cases} 1 + 2x & for\ x \le 0 \\ 1 - 3x & x > 0 \end{cases}$ at x=0

b) Find the nth derivative of $\frac{x+3}{(x-1)(x+2)}$ (3+3)

2. If $y = \sin^{-1} x$ prove that $(1 - x^2)y_{n+2} - (2n+1)xy_{n+1} - n^2 y_n = 0$
3. Prove that a function which is a continuous in a closed interval takes every value between its bounds atleast once .
4. State and prove Lagrange Mean Value Theorem.

5. a) Expand $e^x siny$ in powers of x and y up to second degree terms using Maclaurin's expansion.

   b) Evaluate using L' Hospital's rule $\lim_{x\to 0}(cosx)^{\frac{1}{x^2}}$ (3+3)

6. a) If u = (x - y)^4 + (x+y)^4 find $\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}$

   b) If $u = x^2,\ v = y^2$ find $J' = \frac{\partial(x,y)}{\partial(u,v)}$ (3+3)

7. If $u = \tan^{-1}\left(\frac{x^3+y^3}{x-y}\right)$, x$\neq y$, Show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = sin2u$
8. Test for maximum and minimum of the function $f(x,y) = 2x^2 - xy + y^2 + 7x$